Spatial frequency analysis in vision

from Oliver Braddick

READERS of *Nature* will probably be aware that many experimenters and theorists in visual perception and physiology have taken to using the language of Fourier analysis: they test vision with sine wave grating patterns and discuss the behaviour of the visual system in terms of spatial frequency response. Why has this approach become popular, and has it been justified as enhancing our understanding of vision?

Descriptions in terms of frequency are most familiar when speaking of periodic events in time, such as sound or radio waves, but they may be used for spatial patterns in an analogous way. The number of repetitions of a pattern within a unit of distance is its spatial frequency. The mathematics of Fourier analysis which allow any complex sound to be described as a set of harmonics also allow any visual pattern to be described as a sum of spatial frequency components, that is, patterns whose luminance varies across space as a sine wave. A system designed to transmit time-varying signals, such as a hi-fi set, commonly has its performance specified by its frequency response. Similarly, how well a system transmits spatial patterns can be characterised by its response to different spatial frequencies. In optics, this has been a commonplace technique for evaluating lens systems for many years. The performance of the human eye, however, is determined not only by its optical quality in forming the retinal image, but also by how well the visual system signals the information in that image. It therefore seemed natural to extend the idea of measuring spatial frequency response to the whole system[1,2]. This can be done if we make the fairly uncontentious assumption that for a human subject trying to detect a low-contrast grating pattern, a low threshold indicates a high response of the visual system to that spatial frequency, and so on, using thresholds to measure the contrast sensitivity function.

The spatial frequency response of some component within the visual system may also be measurable: this can be done directly in physiological experiments by measuring the response of single nerve cells to gratings of various frequencies, and indirectly in psychophysical experiments by testing whether two gratings interact in their effects on perception. For example, if one grating can mask detection of another, it is concluded that they must be processed by a common neural 'channel'[3].

The flood of experimental work in what has been dubbed 'gratingology' has come largely from the idea proposed by Campbell and Robson[4] that Fourier methods were valuable, not just for characterising the performance of the whole visual system, but also for understanding the way the system is divided into parallel information-carrying channels. It is generally believed that the properties of these channels define the 'language' in which visual information is conveyed to, and perhaps within, the brain. The idea that this language is best analysed in terms of spatial frequency has been vigorously asserted and contested. If we are to avoid this controversy obscuring the significance of the experimental results, we must keep clear the distinctions between various levels at which spatial frequency methods and Fourier theory may be applied to vision.

First, the spatial frequency response of the visual system can be used as a valuable descriptive measure, without commitment to any particular assumptions about the properties of the visual system. Second, insofar as the visual system, or some component of it, can be treated as linear, the spatial frequency response has important predictive power: since any pattern can be considered as the sum of sine wave components, the visual response to any pattern can be predicted by summing the responses to the separate components. Third, it has been proposed that the visual system at some level consists of parallel, independent channels, each responding to its own limited band of spatial frequency. If so, the visual system itself is performing, at least crudely, an analysis of the visual pattern into its frequency components. Fourth, and most radically, it is possible that this division of the visual pathway into spatial frequency channels might indicate a fundamental way in which visual information is organized: that is, perceptual processes such as selective attention, or recognition, might operate on representations of the visual world in terms of spatial frequency components rather than the more familiar kind of representation in terms of spatial location.

The descriptive use of spatial frequency analysis, at its simplest, amounts to saying that it is important to know not only how well vision conveys very fine detail (high spatial frequencies), but also how well it conveys information about the patterning of light and dark in the image at every scale from the very coarse to the very fine. One application of this is clinical: the quality of vision is conventionally measured in terms of acuity (that is, detail resolution) but some patients complain of poor vision even when their measured acuity is normal. In many cases they can be shown to have abnormally low sensitivity to spatial frequencies below the acuity limit. Indeed, differences in the pattern of contrast sensitivity may be diagnostically significant[5]. An important side effect of the descriptive use of spatial frequency response has been to emphasise the usefulness of contrast as a parameter of vision. Even for non-sine wave stimuli, contrast threshold is often a more useful measure than luminance threshold, since contrast can be varied without introducing the radical changes that occur in the visual system with adaptation to different light levels.

If spatial frequency response is found to have predictive as well as descriptive power, that implies that the visual process concerned shows linear spatial summation. This in turn implies that it could be described equally completely in spatial as in spatial frequency terms. This is true, for instance, of one class of retinal ganglion cells (X-cells)[6]: their response to a pattern can be predicted either from the summation of response to its spatial frequency components, or from summation of the local responses given by each point in the receptive field. The spatial sensitivity profile of the receptive field and the spatial frequency response function may be

Oliver Braddick is in the Department of Experimental Psychology, University of Cambridge.

0028-0836/81/190009-03$01.00

mathematically transformed into each other. In experimental practice, this complete description may be more readily achievable by testing the spatial frequency response, since the cell's response to a grating which covers the whole receptive field is stronger and clearer than that to illumination of a small spot at some point within it.

It can be argued, though, that the spatial frequency approach has a particular value when the requirement of linearity is *not* met. This is because frequency analysis provides tests that very clearly indicate the presence and nature of any non-linearity present. Thus, it was through the investigation of response to sinusoidal stimuli that an important class of non-linear (Y) retinal ganglion cells was discovered by Enroth-Cugell and Robson[6], and that the nature of the mechanisms which produce their characteristic behaviour has begun to be understood[7].

Taking the performance of the visual system as a whole, there are clearly various significant non-linearities, but for the conditions under which many detection experiments are done linear predictions work quite well[4]. This means that it is useful to consider the separate visual response to each of the frequency components making up a complex pattern. Using this mathematical analysis does not require us to suppose that the visual system breaks up the pattern into these components in any physical sense. The frequency response can be taken to describe the properties of visual system as a homogeneous linear spatial filter — the 'single channel' model. The alternative is a 'multi-channel' model in which the analysis into spatial frequency components has more physical reality. In this model, the overall frequency response is the envelope of the sensitivity functions of many independent channels, and different channels may respond to components in a complex pattern. A wide range of psychophysical experiments support the idea of multiple spatial-frequency channels[3,8], although there are still considerable quantitative problems to be resolved on how many parallel channels there are[9], and how narrow is the band of spatial frequencies to which each responds[10].

Physiological experiments, too, show that the visual cortex contains cells responding to different bands of spatial frequencies in the same region of the visual field[11]. It is true that, insofar as these cells show linear spatial summation (as many of them do), their receptive field sensitivity profiles equally describe their properties[12]. For fields of similar form, the differences in optimal frequency will correspond to differences in receptive field size. Even so, the frequency description has a good deal of heuristic value. For instance, a qualitative account in terms of receptive field dimension might lead one to expect that such cells would differ strikingly in the width of bright (or dark) bar to which they respond best. In fact, this turns out to characterise their differences much less clearly than does their spatial frequency response[13].

However, spatial frequency response alone does not give all the information expressed in a spatial sensitivity profile. This is because the response is a function of the phase as well as the spatial frequency of a grating. A cell which responds well to bar patterns will show optimal response to different grating frequencies when they have a peak of the waveform in a given position, while responsiveness to edge patterns corresponds to the requirement that waveforms of different frequency should have their maximum gradient in the same position. What is apparent from making the distinction in terms of phase, rather than edges versus bars, is that any pattern can be considered a mixture of the two phase relationships. Both psychophysicists and physiologists[12] are now exploring the possibility that the division into two types of phase response is another form of 'channeling' for encoding spatial patterns in vision.

Given the existence of channels that respond independently to different spatial frequency bands, should we conclude that visual perception operates in the Fourier domain? In modern digital image processing, it is often found useful to perform various operations on an array of, say 256×256 spatial frequencies, each of which is derived in the Fourier transform of an entire picture of 256×256 local elements. It is most unlikely that visual perception works in anything like this way. Physiological and psychophysical evidence concur that, at the levels we have been considering, analysis is fairly local: receptive fields are limited in size. There is a mathematical inverse relationship between the width of the spatial frequency band to which a channel responds and the range of spatial positions to which it responds[14]. It appears that the visual system represents a compromise, dividing the spatial frequency spectrum fairly coarsely within local, but not very small, areas.

In fact, it would probably be pointless for the visual system to measure spatial frequency components, or anything else, over the visual field as a whole. What we usually want to recognise is not the visual field as a unified entity, but objects within it. Could we be recognising an object by means of the spatial frequency components detected by a local set of channels? In most cases, this seems quantitatively unlikely. Analyses of spatial frequency channels in central vision have generally found them only for spatial frequencies above 1 cycle/degree. Such channels could encode an adequate frequency spectrum only for an object that was roughly ½ degree across, or smaller. With the important exception of written characters, we need to recognise most objects over a range of visual angles markedly larger than this. A similar conclusion arises from physiological receptive field sizes, and from the extent of visual field analysed by a single cortical 'hypercolumn'[15]: at the level of visual cortex, a single object will normally be analysed by many, separate, receptive fields. An important aspect of the processing of visual shape must be the 'grouping' of information from local analysers spread out over an area of the visual field.

What, then, might be the role of spatial frequency analysis in the process of perception? Marr and Hildreth[16] have suggested that coincident signals in different frequency channels provide an effective criterion for finding edges in the visual field. In this view spatial frequency analysis is used at a rather low level to define features, which themselves are part of a representation in the domain of space rather than spatial frequency. Separate processing of spatial frequency bands has been proposed as playing a rather similar role of local disambiguation in the computation of stereo disparity[17,18]. In these models, the information signalled by spatial frequency channels is not separately available for higher perceptual processes. This argument has been supported by experimental evidence that patterns can differ quite markedly in the Fourier domain without it being possible to use these differences for perceptual grouping[19] (although this result concerns the ability to separate componets that differ in orientation, not in spatial frequency itself).

On the other hand, one of the best known effects in grating psychophysics is the 'perceived spatial frequency shift' produced by adaptation to a grating[20]. This

1. Schade, O.H. *J. opt. Soc. Am.* **46,** 721 (1956).
2. Campbell, F.W. & Green, D.G. *J. Physiol., Lond.* **181,** 576 (1965).
3. Braddick, O., Campbell, F.W. & Atkinson, J. in *Handbook of Sensory Physiology, Vol. VIII: Perception* (eds Held, R., Leibowitz, H. & Teuber, H.L.) (Springer, Heidelberg, 1978).
4. Campbell, F.W. & Robson, J.G. *J. Physiol., London.* **197,** 551 (1968).
5. Arden, G.B. *Br. J. Ophthalmol.* **62,** 198 (1978). Bodis-Wollner, I. & Diamond, S.P. *Brain* **99,** 695 (1976). Hess, R.F. & Howell, E.R. *Vision Res.* **17,** 1049 (1977).
6. Enroth-Cugell, C. & Robson, J.G. *J. Physiol., Lond.* **187,** 517 (1966).
7. Hochstein, S. & Shapley, R.M. *J. Physiol., Lond.* **262,** 237 (1976).
8. Blakemore, C. & Campbell, F.W. *J. Physiol., Lond.* **203,** 237 (1969). Graham, N. & Nachmias, J. *Vision Res.* **11,** 251 (1971). Graham, N., Robson, J.G. & Nachmias, J. *Vision Res.* **18,** 815 (1978). Sachs, M., Nachmias, J. & Robson, J.G. *J. opt. Soc. Am.* **61,** 1176 (1971).
9. Wilson, H.R. & Bergen, J.R. *Vision Res.* **19,** 19 (1979). Watson, A.B. *Vision Res.* (in the press).
10. Graham, N. *Vision Res.,* **17,** 637 (1977). King-Smith, P.E. & Kulikowski, J.J. *Vision Res.* **21,** 235 (1981).
11. Maffie, L. & Fiorentini, A. *Vision Res.* **13** 1225 (1973).
12. Movshon, J.A., Thompson, I.D. & Tolhurst, D.J. *J. Physiol., Lond.* **283,** 53 (1978).
13. De Valois, R.L., Albrecht, D.G. & Thorell, L.G. in *Frontiers in Visual Science* (eds Cool, S.J. & Smith, E.L. III) (Springer, New York, 1977).
14. Mackay, D.M. *Nature* **289, 117 (1981).**
15. Hubel, D.H. & Wiesel, T.N. *Proc. R. Soc. Lond.* **B198** (1977).
16. Marr, D. & Hildreth, E. *Proc R. Soc. Lond.* **B207,** 187 (1980).
17. Marr, D. & Poggio, T. *Proc.. R. Soc. Lond.* **B204,** 301 (1979).
18. Mayhew, J.W.W. & Frisby, J. *Perception* **9,** 60 (1980).
19. Mayhew, J.E.W. & Frisby, J.P. *Nature* **275,** 438 (1978).
20. Blakemore, C., Sutton, P. & Nachmias, J. *J. Physiol., Lond.* **210,** 727 (1970).
21. Campbell, F.W. & Marr, D. Discussion in *Phil. Tans. R. Soc.* **B209,** 9 (1980).

0028-0836/81/190010-02$01.00

effect is best explained by the assumption that perceived spacing of the grating is directly determined by which spatial frequency channel responds most strongly. It can be argued that judging the spatial frequency of a grating is more related to recognising a repetitive texture, in which we are unconcerned with the spatial identity of individual features, than to perceiving the size and shape of a discrete object. Maybe objects are represented in spatial terms, but the textures of their surfaces in spatial frequency terms.

Speaking of 'objects' should not lead us into the assumption that there we divide any particular visual scene in a fixed way. Clearly for some purposes we are primarily concerned with the gross layout of our visual environment, while for others we need information that is present in fine detail. It is tempting to suppose that for different perceptual tasks we might switch our attention to different spatial frequency bands. However, it is hard to find an experimental example that demands an explanation as spatial frequency tuning of attention, and there are certainly instances where we do *not* find it possible to select a frequency band in this way[21].

Many visual scientists who are sceptical or open minded on how far higher perceptual processes speak the language of spatial frequency nonetheless find that language invaluable for describing and manipulating their experimental stimuli and the information signalled in the visual system. New insights come from thinking of the visual image as containing patterns of light and dark over a range of spatial scales, or 'degrees of blurredness'. Formally, this idea, and analyses that embody it, may be expressible as exactly in spatial as in spatial frequency terms. The fact remains that by exploring the implications of the spatial frequency description, and by using stimulus patterns which were simple to describe that way, we have learned a great deal about vision and by all indications have more to learn yet. □

Sliding charge density waves

from P. A. Lee

THE idea of a charge density wave transition goes back to Peierls[1] and, independently, to Fröhlich[2] in 1954. They reasoned that a one-dimensional metal can lower its energy by distorting the lattice and forming a gap at the Fermi surface, thereby making a transition into an insulating state. The term 'charge density wave' refers to the fact that the lattice and the electron charge density form a new periodic structure, with a wavelength λ that is longer than the original lattice period a. Fröhlich further reasoned that if the charge density wave is incommensurate, that is, a/λ is not a simple rational fraction like 1/2 or 2/3, then the entire charge density wave structure can slide through the lattice, thereby contributing to the electrical conductivity. Since these ideas were originally formulated in terms of one-dimensional systems, for many years they were considered with nothing more than theoretical curiosity.

In the early 1970s the interests of many physicists turned to quasi one- and two-dimensional systems. The existence of a charge density wave ground state in these systems turns out to be the rule rather than the exception, and a large number of examples have been discovered. Experimentally the onset of the charge density wave is signaled by a rise in the resistivity at some temperature T_c. In quasi one-dimensional systems a transition to an insulating state often occurs, whereas in the layered compounds the system often stays metallic. The most direct microscopic evidence for a charge density wave transition is the appearance of new superlattice spots associated with the new lattice periodicity λ which is detected by X-ray, neutron or electron diffraction. Prominent examples of charge density wave systems are the organic charge transfer salts[3], such as tetrathiafulvalene-tetracyanoquinodimethane (TTF-TCNQ) and layered compounds[4] formed from transition metals and chalcogens, such as TaS_2 and $NbSe_2$.

However, until 1976, none of the charge density wave systems had exhibited the sliding conductivity envisioned by Fröhlich. This is because the charge density wave was either commensurate or pinned by impurities. Impurities are sensitive to the charge density oscillations and can lock the charge density wave in place, a phenomenon very analogous to static friction[5]. As in friction, a sufficiently large force should dislodge the charge density wave. Thus the observation[6,7] in 1976 of nonlinear electrical conductivity in $NbSe_3$ was greeted with a good deal of interest. In $NbSe_3$ there are two transitions at 59K and 144K; below each of these temperatures the resistivity rises. It was found that beyond a certain threshold electric field[8], which can be as low as a few meV per cm for the low-temperature transition and 0.1 eV per cm for the higher-temperature transition, the resistivity becomes nonlinear and begins to drop, until it saturates to a value close to that expected if the charge density wave transition had not occurred.

The conductivity is also found to be highly frequency dependent, rising with increasing frequency until it coincides with the high field d.c. value at ~100 MHz (refs 7 and 9). It soon became clear that the electric field is not destroying the charge density wave, because an X-ray diffraction experiment shows that the superlattice spots are not affected by a current through the sample even when the current exceeds the linear threshold[10]. Diffraction experiments reveal incommensurate lattice distortions with periods λ such that $a/\lambda = 0.24$ and 0.26 for the upper and lower transitions. These data are consistent with an interpretation in terms of impurity pinning of the charge density wave[11]. The charge density wave can undergo small oscillation, thus accounting for the frequency-dependent conductivity. The pinning frequency is very low, possibly because the impurity concentration in $NbSe_3$ is very small. What makes $NbSe_3$ special among charge density wave systems in this respect is not entirely understood at present. The weakness of the pinning also manifests itself in the low-frequency dielectric constant, which has been measured to be greater than 10^8 (see ref. 9). Presumably a small electric field is sufficient to overcome the weak pinning so that the charge density wave can slide in the way envisioned by Fröhlich — an interpretation supported by an experiment in which impurities were introduced into $NbSe_3$ and the threshold electric field was found to increase[12]. Instead of classical depinning, an alternative mechanism involving the quantum mechanical tunneling of macroscopic segments (up to several microns in length) of charge density waves has been proposed[13], but all existing theories are based on the collective motion of the charge density wave ground state described by Fröhlich.

Further study of $NbSe_3$ yielded more surprises. When the current exceeds the threshold, voltage noise across the sample greatly increases. Superimposed on this background noise are periodic components with well defined frequencies[8], which scale linearly with the excess nonlinear current[14] and are typically in the range of kHz to MHz. A model for the noise is that it arises when the charge density wave moves through the impurities or the lattice, much like a particle rolling down a washboard. If certain assumptions are made about the charge being carried by the sliding charge

P.A. Lee is at the Bell Laboratories, Murray Hill, New Jersey.

1. Peierls, R. E. *Quantum Theory of Solids* (Oxford University Press, London, 1955).
2. Fröhlich, H. *Proc. R. Soc.* **A223**, 296 (1954).
3. *The Physics and Chemistry of Low Dimensional Solid* (ed. Alcacer, L., 1979).
4. Wilson, J. A. *et al. Adv. Phys.* **24**, 117 (1975).
5. Lee, P. A. *et al. Solid State Commun.* **14**, 703 (1974).
6. Monceau, P. *et al. Phys. Rev. Lett.* **37**, 602 (1976).
7. Ong, N. P. & Monceau, P. *Phys. Rev.* **B16**, 3443 (1977).
8. Flemming, R. M. & Grimes, C. C. *Phys. Rev. Lett.* **42**, 1423 (1979).
9. Grüner, G. *et al. Phys. Rev. Lett.* **45**, 935 (1980).
10. Fleming, R. M. *et al. Phys. Rev.* **B18**, 5560 (1978).
11. Lee, P. A. & Rice, T. M. *Phys. Rev.* **B19**, 3970 (1979).
12. Ong, N. P. *et al. Phys. Rev. Lett.* **42**, 811 (1979).
13. Bardeen, J. *Phys. Rev. Lett.* **45**, 1978 (1980).
14. Monceau, P. *et al. Phys. Rev. Lett.* **45**, 43 (1980).
15. Fung, K. K. & Steeds, J. W. *Phys. Rev. Lett.* **45**, 1696 (1980).

0028-0836/81/190011-02$01.00